

Feel free to print & annotate on this worksheet!
Calculator is allowed for this worksheet.
You must write down all stages of your working.
(Answers to each question are at the end of the worksheet 😊)

1. A curve C has equation

$$y = x^5 + 3x - 6\sqrt{x}$$

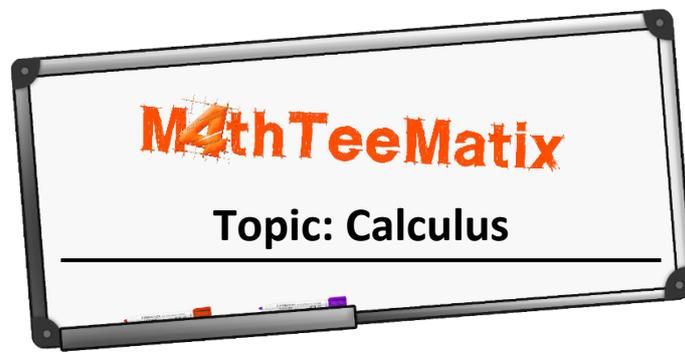
Find

a) $\frac{dy}{dx}$

b) $\frac{d^2y}{dx^2}$

2. Write the gradient function for

$$y = \sin(4x^2)$$



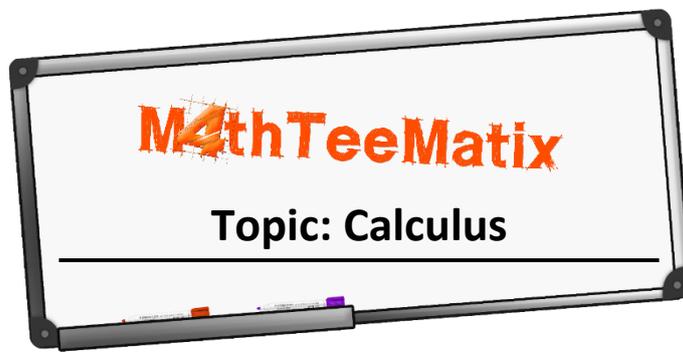
3. Integrate the function

$$f(x) = 3x^4 - x^{\frac{1}{2}}$$

4. A curve C has equation

$$y = x^3 + 2x^2 - 7x - 20$$

Find the stationary points for curve C



5. Curve C has equation

$$y = -x^4 + 2x^2 + 7x$$

A region R is bounded by the curve, the x -axis, and the lines $x = 1$ and $x = 2$
(You may sketch the graph if you find it more helpful)

Use integration to find the area of R

Answers...(in green)

1. A curve C has equation

$$y = x^5 + 3x - 6\sqrt{x}$$

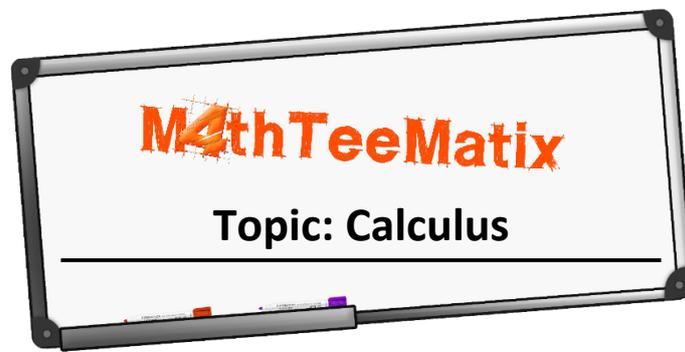
Find

a. $\frac{dy}{dx}$

$$y = x^5 + 3x - 6x^{1/2}$$

$$\frac{dy}{dx} = 5x^{5-1} + 3x^{1-1} - 6\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$\frac{dy}{dx} = 5x^4 + 3 - 3x^{-1/2}$$



b. $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 5x^4 + 3 - 3x^{-\frac{1}{2}}$$
$$\frac{d^2y}{dx^2} = 5(4)x^{4-1} - 3\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

$$\frac{d^2y}{dx^2} = 20x^3 + \frac{3}{2}x^{-\frac{3}{2}}$$

2. Write the gradient function for

$$y = \sin(4x^2)$$

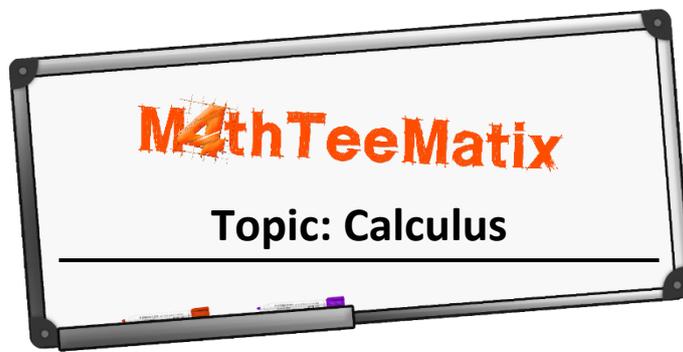
$$\frac{dy}{dx} = 8x \cos(4x^2)$$

3. Integrate the function

$$f(x) = 3x^4 - x^{\frac{1}{2}}$$

$$\int 3x^4 - x^{\frac{1}{2}} dx$$
$$\Rightarrow \frac{3x^{4+1}}{4+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$
$$\Rightarrow \frac{3}{5}x^5 - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\Rightarrow \frac{3}{5}x^5 - \frac{2}{3}x^{\frac{3}{2}} + C$$



4. A curve C has equation

$$y = x^3 + 2x^2 - 7x - 20$$

Find the stationary points for curve C

$$\begin{aligned}\frac{dy}{dx} &= 3x^2 + 4x - 7 \\ 0 &= 3x^2 + 4x - 7 \\ &= 3x^2 - 3x + 7x - 7 \\ &= 3x(x-1) + 7(x-1) \\ &= (3x+7)(x-1) \\ x &= -\frac{7}{3} \text{ or } 1\end{aligned}$$

when $x = 1$

$$y = 1^3 + 2(1)^2 - 7(1) - 20 = -24$$

when $x = -\frac{7}{3}$

$$y = \left(-\frac{7}{3}\right)^3 + 2\left(-\frac{7}{3}\right)^2 - 7\left(-\frac{7}{3}\right) - 20 = -21.03\bar{7}$$

∴ stationary points are $(1, -24)$ & $\left(-\frac{7}{3}, -21.03\bar{7}\right)$

5. Curve C has equation

$$y = -x^4 + 2x^2 + 7x$$

A region R is bounded by the curve, the x -axis, and the lines $x = 1$ and $x = 2$
(You may sketch the graph if you find it more helpful)

Use integration to find the area of R

$$\begin{aligned}&\int_1^2 (-x^4 + 2x^2 + 7x) dx \\ &\left[\frac{-x^5}{5} + \frac{2x^3}{3} + \frac{7x^2}{2} \right]_1^2 \\ &\left(\frac{-(2)^5}{5} + \frac{2(2)^3}{3} + \frac{7(2)^2}{2} \right) - \left(\frac{-(1)^5}{5} + \frac{2(1)^3}{3} + \frac{7(1)^2}{2} \right) \\ &12.9333\dots - 3.9666\dots\end{aligned}$$

Area of $R = \frac{269}{30}$ or 8.97

End of worksheet