MathTeeMatix

Topic: Calculus

Feel free to print & annotate on this worksheet!
Calculator is allowed for this worksheet.
You must write down all stages of your working.
(Answers to each question are at the end of the worksheet 😊)

1. A curve C has equation

$$y = x^5 + 3x - 6\sqrt{x}$$

Find

a) $\frac{dy}{dx}$

b) $\frac{d^2y}{dx^2}$

2. Write the gradient function for

$$y = \sin(4x^2)$$

MathTeeMatix

Topic: Calculus

3. Integrate the function

$$f(x) = 3x^4 - x^{\frac{1}{2}}$$

---

4. A curve C has equation

$$y = x^3 + 2x^2 - 7x - 20$$

Find the stationary points for curve C

MathTeeMatix

Topic: Calculus

5. Curve C has equation

$$y = -x^4 + 2x^2 + 7x$$

A region R is bounded by the curve, the x-axis, and the lines $x = 1$ and $x = 2$
(You may sketch the graph if you find it more helpful)

Use integration to find the area of R

Answers…(in green)

1. A curve C has equation

$$y = x^5 + 3x - 6\sqrt{x}$$

Find

a. $\frac{dy}{dx}$

$y = x^5 + 3x - 6x^{\frac{1}{2}}$

$\frac{dy}{dx} = 5x^{5-1} + 3x^{1-1} - 6\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$

$$\frac{dy}{dx} = 5x^4 + 3 - 3x^{-\frac{1}{2}}$$

MathTeeMatix

Topic: Calculus

b. $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 5x^4 + 3 - 3x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = 5(4)x^{4-1} - 3\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

$$\boxed{\frac{d^2y}{dx^2} = 20x^3 + \frac{3}{2}x^{-\frac{3}{2}}}$$

---

2. Write the gradient function for

$$y = \sin(4x^2)$$

$$\boxed{\frac{dy}{dx} = 8x\cos(4x^2)}$$

---

3. Integrate the function

$$f(x) = 3x^4 - x^{\frac{1}{2}}$$

$$\int 3x^4 - x^{\frac{1}{2}}\,dx$$

$$\Rightarrow \frac{3x^{4+1}}{4+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$\Rightarrow \frac{3}{5}x^5 - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\boxed{\Rightarrow \frac{3}{5}x^5 - \frac{2}{3}x^{\frac{3}{2}} + C}$$

MathTeeMatix

Topic: Calculus

4. A curve C has equation

$$y = x^3 + 2x^2 - 7x - 20$$

Find the stationary points for curve C

$\frac{dy}{dx} = 3x^2 + 4x - 7$

$0 = 3x^2 + 4x - 7$

$= 3x^2 - 3x + 7x - 7$

$= 3x(x-1) + 7(x-1)$

$= (3x+7)(x-1)$

$x = -\frac{7}{3}$ or 1

when $x = 1$

$y = 1^3 + 2(1)^2 - 7(1) - 20$

$= -24$

when $x = -\frac{7}{3}$

$y = (-\frac{7}{3})^3 + 2(-\frac{7}{3})^2 - 7(-\frac{7}{3}) - 20$

$= -21.037$

∴ stationary points are $(1, -24)$ & $(-\frac{7}{3}, -21)$

---

5. Curve C has equation

$$y = -x^4 + 2x^2 + 7x$$

A region R is bounded by the curve, the x-axis, and the lines $x = 1$ and $x = 2$
(You may sketch the graph if you find it more helpful)

Use integration to find the area of R

$\int_1^2 -x^4 + 2x^2 + 7x \, dx$

$\left[\frac{-x^5}{5} + \frac{2x^3}{3} + \frac{7x^2}{2}\right]_1^2$

$\left(\frac{-(2)^5}{5} + \frac{2(2)^3}{3} + \frac{7(2)^2}{2}\right) - \left(\frac{-(1)^5}{5} + \frac{2(1)^3}{3} + \frac{7(2)^2}{2}\right)$

$12.9333\ldots \quad - \quad 3.9666\ldots$

Area of $R = \frac{269}{30}$ or 8.97

End of worksheet